Stochastic growth models for driven interfaces through random media in two and three dimensions

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We introduce two simple stochastic growth models which describe the motion of the interfaces driven through random media in two and three dimensions. One model describes the motion of the interface driven through isotropic random media, where the dynamics of the interface can be described by the quenched Edwards-Wilkinson (QEW) equation. The other model describes the motion of the interface driven through anisotropic random media, where the dynamics of the interface can be described by the quenched Kardar-Parisi-Zhang (QKPZ) equation. We show via computer simulations that two models belong to the QEW and QKPZ universality class in two and three dimensions, respectively.

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The dynamics of a driven interface in random media has attracted much attention during the last decade because it is relevant to various interesting phenomena [1–5]. The driven motion of the interface in random media takes place due to the interplay between the resistance force by quenched disorder in random media and the driving force acting on the interface. The interface is pinned if driving force F is smaller than the pinning strength induced by the quenched disorder, and moves with a constant velocity for F greater than the pinning strength. Hence, there exists a threshold of the driving force F_c separating two regimes. This phenomenon is called the pinning-depinning transition.

Near the depinning threshold, dynamics of a driven interface in random media can be described in terms of the roughness exponent α and the growth exponent β [6,7]. The two exponents are defined from the interface width $W(L,t) = \langle L^{-d}\Sigma_i[h_i(t)-\overline{h}(t)]^2\rangle^{1/2}$, which scales as L^{α} for a long time ($t \gg L^z$) and t^{β} at the early stages ($t \ll L^z$) of the process. $z=\alpha/\beta$ is called the dynamic exponent. Here $\overline{h}$, L, d, and $h_i(t)$ denote the mean height, system size, substrate dimension, and the height at site i and time t, respectively.

The dynamics of the driven interface in random media can be explained well by a Langevin-type continuum equation. A well-known equation describing the motion of a driven interface in random media is the quenched Kardar-Parisi-Zhang (QKPZ) equation [7]:

$$\frac{\partial h(\mathbf{x},t)}{\partial t} = \nu \nabla^2 h + \frac{\lambda}{2}(\nabla h)^2 + F + \eta(\mathbf{x},h), \tag{1}$$

where the quenched noise $\eta(x,h)$ satisfies $\langle \eta(\mathbf{x},h)\rangle = 0$ and $\langle \eta(\mathbf{x},h)\,\eta(\mathbf{x}',h')\rangle = 2D\,\delta^d(\mathbf{x}-\mathbf{x}')\,\delta(h-h')$.

Many studies have been carried out to describe and understand the motion of the driven interface following the QKPZ equation. Tang and Leshhorn [8] suggested that the directed percolation depinning (DPD) model [9] belongs to the QKPZ universality class. They argued that the roughness exponent α in the QKPZ universality class is given by the ratio of two correlation length exponents, $\nu_{\perp}$ and $\nu_{\parallel}$, in the perpendicular and parallel directions of directed percolating clusters, which is $\alpha = \nu_{\perp}/\nu_{\parallel} \approx 0.63$ in one dimension. Leschhorn [10] also showed that the roughness exponent in the QKPZ universality class is $\alpha \approx 0.63$ in one dimension via the numerical integration of the QKPZ equation and the automaton model, which is the discrete version of the QKPZ equation. In two and three dimensions, the DPD model gives $\alpha \simeq 0.48$ [11] and $\alpha \simeq 0.38$ [12], respectively. A self-organized growth model in the QKPZ universality class was also introduced by Sneppen [13].

When there is no nonlinear term $[\lambda(\nabla h)^2]$ in Eq. (1), the equation is called the quenched Edwards-Wilkinson (QEW) equation. The dynamic scaling behavior of the QEW equation is completely different from that of the QKPZ equation. Analytical studies [14] of the QEW equation predict $\alpha = (4-d)/3$ and $z = 2-(2/9)(4-d)$, whereas the direct integration of the QEW equation [15] and the computer simulation of the automaton model [16], which is the discrete version of this equation, gives $\alpha \approx 1.25$ in one dimension.

Thus, there exist two distinct universality classes characterizing dynamics of driven interfaces in random media depending on whether there exists the KPZ nonlinear term in Eq. (1). Amaral *et al.* [17] introduced a method to classify the two distinct universality classes from computer simulations by analyzing the dependence of the interface velocity $v(m)$ on the slope m of the tilted substrate from the computer simulation. In the case of the QEW class, the slope dependence of the interface velocity is either absent or vanishes at the depinning threshold, which indicates that the KPZ nonlinear term does not exist at the threshold. In the case of the QKPZ class, however, the growing velocity $v(m)$ depends on the slope m near the depinning threshold. It indicates that the KPZ nonlinearity exists even at the depinning threshold, i.e., $v=0$. Tang *et al.* [18] and Park *et al.* [19] showed that the KPZ nonlinearity is originated from the anisotropic properties of the random medium. The KPZ nonlinearity effect does not exist in the motion of interfaces driven through isotropic random media. Tang *et al.* argued that the KPZ nonlinear term can be generated from anisotropic properties of random media. If the interface under anisotropic random media is driven by external force, then the threshold F_c depends on the slope m of the tilted substrate. The dependence of the threshold F_c on m can be written as

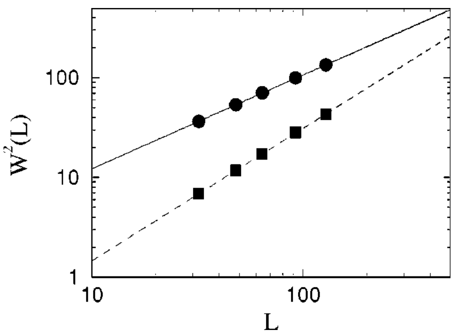

FIG. 1. The plots of width $W^2(L)$ versus the system size L in two dimensions for model A (the filled circle) and model B (the filled square). The solid guideline represents $\alpha \simeq 0.47$ and the dotted line shows $\alpha \simeq 0.66$ in two dimensions.

$$F_c(m) - F_c(0) \propto -|m|^{1/\nu(1-\alpha)}, \tag{2}$$

where the correlation length ξ parallel to the interface scales as $\xi \sim (F - F_c)^{\nu}$ and ν is called the correlation length exponent. The KPZ nonlinearity in the driven interface is originated from the characteristics of the medium rather than from the kinematic effects [18]. We confirmed the TKD's suggestion by directly controlling the degree of the anisotropy of random media [19]. In spite of a lot of numerical works, there are no simple self-organized stochastic growth models for the QEW and the QKPZ universality class in two or three dimensions. Therefore, it would be worth introducing simple growth models describing the motion of interfaces driven in random media in high dimensions.

In this paper, we introduce two kinds of stochastic growth models which describe the motion of the driven interfaces in random media in two and three dimensions. In our models, we use the Family growth rule [20], which is related to the Edwards-Wilkinson (EW) [21] universality class occurring in the interface growth in homogeneous media. Although one uses the same growth rule, dynamical behavior of driven interfaces can be changed drastically according to the way of updating random numbers on the interface each time, where random numbers represent the impurities in random media. We find that our models belong to the QEW and the QKPZ universality class by using different updating rules of the random numbers in the two models.

The growth rule of our model (we call it model A) is defined as follows: First, we preassign random numbers between 0 and 1 representing impurities in random media, to all perimeter sites of the initially flat substrate. A particle is deposited on the site i with the lowest minimum random number on the interface. The deposited particle allows to diffuse to a nearest-neighbor site with the smallest height if it finds lower height than h_i. When the heights of all nearest-neighbor sites are the same and are smaller than h_i, the particle moves to a randomly chosen one of its nearest-neighbor sites. Then we update the random number at the newly occupied site.

Our simulations were carried out starting from a flat initial surface with periodic boundary conditions in two and three dimensions. Numerical data were averaged over more than 100 configurations. Figure 1 shows the plot of the surface width $W^2(L)$ versus system size L with $L^2 = 32^2$, 48^2, 64^2, 92^2, and 128^2 in two dimensions. The filled circles represent the data obtained from the model A. The solid guideline represents that $\alpha \simeq 0.47$. This value agrees with that obtained from the DPD model in the QKPZ universality class. In this model, however, the saturation is reached so fast that we could not obtain the value of the β exponent.

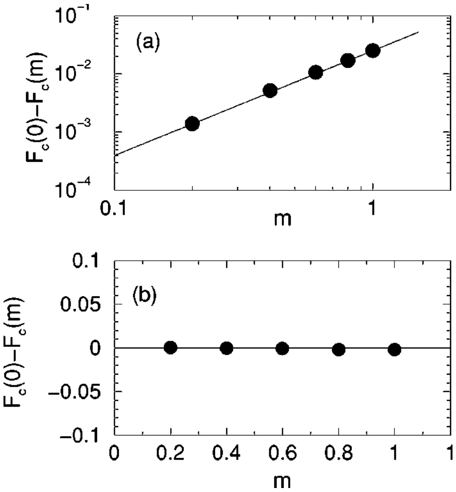

FIG. 2. The plots of the threshold force $F_c(0) - F_c(m)$ as a function of the average orientation of the surface m for model A (a) and for model B (b).

In the growth rule of this model, we always drop a particle at each timestep regardless of the tilt of the substrate. This means that the growth velocity in our model does not depend on the tilt of the substrate, i.e., the KPZ nonlinear term is not generated kinematically in the model A.

To survey the origin of the KPZ nonlinearity in this model, we look into the characteristics of the medium by considering the tilt dependency of the threshold force. If the deposited particle diffuses to its nearest-neighbor site, the random number at the selected site is not changed. The random number at the selected site is the lowest one among the random numbers on the interface before the diffusion occurs. The selected site, therefore, tends to be chosen again at the next timestep because there is great probability that the newly updated random number at the updated site is larger than that at the selected site. That is, if the diffusion process occurs, the selected site is often selected again in the next timestep. It makes the value of the threshold force lower when the diffusion process occurs. The more the substrate is tilted, the more the diffusion process occurs, so that the threshold force depends on the tilt of the substrate [see Fig 2(a)]. This dependence of the threshold force on the tilt of the substrate indicates that the KPZ nonlinear term is generated by the anisotropic property of the medium and thus the model A belongs to the QKPZ universality class.

Next, we consider another model (model B), where the tilt dependency of the threshold force is absent. The dynamic rule in this model is the same as that of model A except for the rule updating random numbers. In order to avoid the tilt dependency of the threshold force, we always update the two

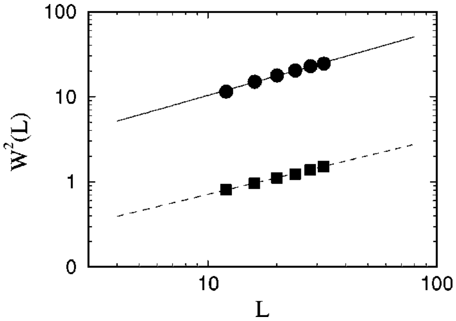

FIG. 3. The plots of width $W^2(L)$ versus the system size L in three dimensions for model A (the filled circle) and model B (the filled square). We obtained $\alpha \simeq 0.38$ and $\alpha \simeq 0.33$ for model A and model B, respectively.

random numbers in this model each time. If diffusion of a newly deposited particle occurs, two random numbers at the selected site and the newly occupied one are updated simultaneously. If the diffusion does not occur, two random numbers at the selected site and at a randomly chosen one of its nearest-neighbor sites are updated simultaneously. By this updating method, the random number at the selected site is always updated regardless of the diffusion process. Therefore the threshold force does not depend on the tilt of the substrate. For this model, the plot of the surface width $W^2(L)$ versus system size L is marked with a filled square in Fig. 1. The dotted line shows $\alpha \approx 0.66$, which is in excellent agreement with the analytic result of the QEW equation in two dimensions.

We also obtained the threshold force F_c by measuring distribution of random numbers and distribution of the minimum random numbers in the critical state [22]. Figure 2 shows the plot of the threshold force $F_c(m)$ versus the tilt m of the substrate for the model A (a) and for the model B (b). The straight guideline in Fig. 2(a) has the slope of 1.80. This value agrees with that expected from $1/\nu(1-\alpha)$ with $\nu = 1.06$ and $\alpha = 0.48$ in the DPD model in two dimensions. Figure 2(b) shows that the threshold force is independent of the tilt of the substrate for model B. These results confirm that model A belongs to the QKPZ universality class and model B to the QEW universality class.

We also studied models A and B in three dimensions. Figure 3 shows the plot of the interface width $W^2(L)$ as a function of the system size L in three dimensions for model A (the filled circle) and model B (the filled square). The solid guideline shows $\alpha \approx 0.38$ and the dotted line represents $\alpha \approx 0.33$. These values suggest that model A belongs to the QKPZ class and model B to the QEW class for three dimensions.

Several years ago, Sneppen introduced two simple self-organized growth models which show two different scaling behaviors when the growth rule is a bit changed like in our models [13]. In the models, the different scaling behaviors originate from the fact that the KPZ nonlinear term, λ, generated in the growth process, has different signs for each growth rule. In one model, the scaling behavior of the model can be explained by the QKPZ equation with $\lambda > 0$, which is in the same universality class as the DPD model giving the roughness exponent $\alpha \simeq 0.63$. In the other model, it can be explained by the QKPZ equation with $\lambda < 0$ in which the roughness exponent is $\alpha \simeq 1$ [23] and thus shows the different scaling behavior from the QKPZ equation with $\lambda > 0$. Meanwhile, two different scaling behaviors in our models originate from the fact about whether the KPZ nonlinear term is generated or not in the growth process. Model A, where the KPZ nonlinearity is generated in the growth process is described by the QKPZ equation with $\lambda > 0$, while the KPZ nonlinearity is not generated in model B, which belongs to the QEW universality class.

In summary, we have introduced two simple stochastic growth models, which belong to the QKPZ and the QEW universality class in two and three dimensions. The same dynamic growth rule has been used in the two models, whereas the updating rule of the random number is different in the two models. This slight modification of the updating algorithm make the two models belong to the different universality classes. For the model A, where the properties of random medium are anisotropic, we obtained the roughness exponents $\alpha \simeq 0.47$ and $\alpha \simeq 0.38$ in two and three dimensions, respectively. These results are in good agreement with the DPD model [11] belonging to the QKPZ universality class. For the model B, where the properties of random medium are isotropic, we obtained the roughness exponents $\alpha \simeq 0.66$ and $\alpha \simeq 0.33$ in two and three dimensions, respectively. These values are in good agreement with the analytic solutions [14] of the QEW equation. Model B is a good model describing the dynamics of the QEW universality class in two and three dimensions.

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